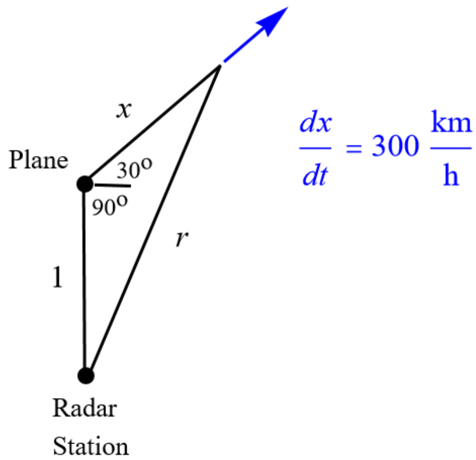
Exercise 47

A plane flying with a constant speed of 300 km/h passes over a ground radar station at an altitude of 1 km and climbs at an angle of 30°. At what rate is the distance from the plane to the radar station increasing a minute later?

Solution

Draw a schematic of the plane's path at a certain time.



The aim is to find dr/dt when $t = 1$ min $= (1/60)$ hour. Start with the formula relating the sides of this triangle, the law of cosines.

$$r^2 = 1^2 + x^2 - 2(1)(x)\cos 120°$$

$$r = \sqrt{1 + x^2 - 2x\left(-\frac{1}{2}\right)}$$

$$= \sqrt{x^2 + x + 1}$$

Take the derivative of both sides with respect to t by using the chain rule.

$$\frac{d}{dt}(r) = \frac{d}{dt}\left(\sqrt{x^2 + x + 1}\right)$$

$$\frac{dr}{dt} = \frac{1}{2}(x^2 + x + 1)^{-1/2} \cdot \frac{d}{dt}(x^2 + x + 1)$$

$$= \frac{1}{2}(x^2 + x + 1)^{-1/2} \cdot \left(2x \cdot \frac{dx}{dt} + \frac{dx}{dt}\right)$$

$$= \frac{1}{2}(x^2 + x + 1)^{-1/2} \cdot (2x + 1)\frac{dx}{dt}$$

$$= \frac{2x + 1}{2\sqrt{x^2 + x + 1}}\frac{dx}{dt}$$

Therefore, when the plane travels a distance, $x = 300 * (1/60)$ kilometers, one minute later, the rate of change of the distance from the plane to the radar station with respect to time is

$$\left.\frac{dr}{dt}\right|_{x=300/60} = \frac{2\left(\frac{300}{60}\right)+1}{2\sqrt{\left(\frac{300}{60}\right)^2+\left(\frac{300}{60}\right)+1}}(300) = \frac{1650}{\sqrt{31}}\ \frac{\text{km}}{\text{h}} \approx 296.349\ \frac{\text{km}}{\text{h}}.$$